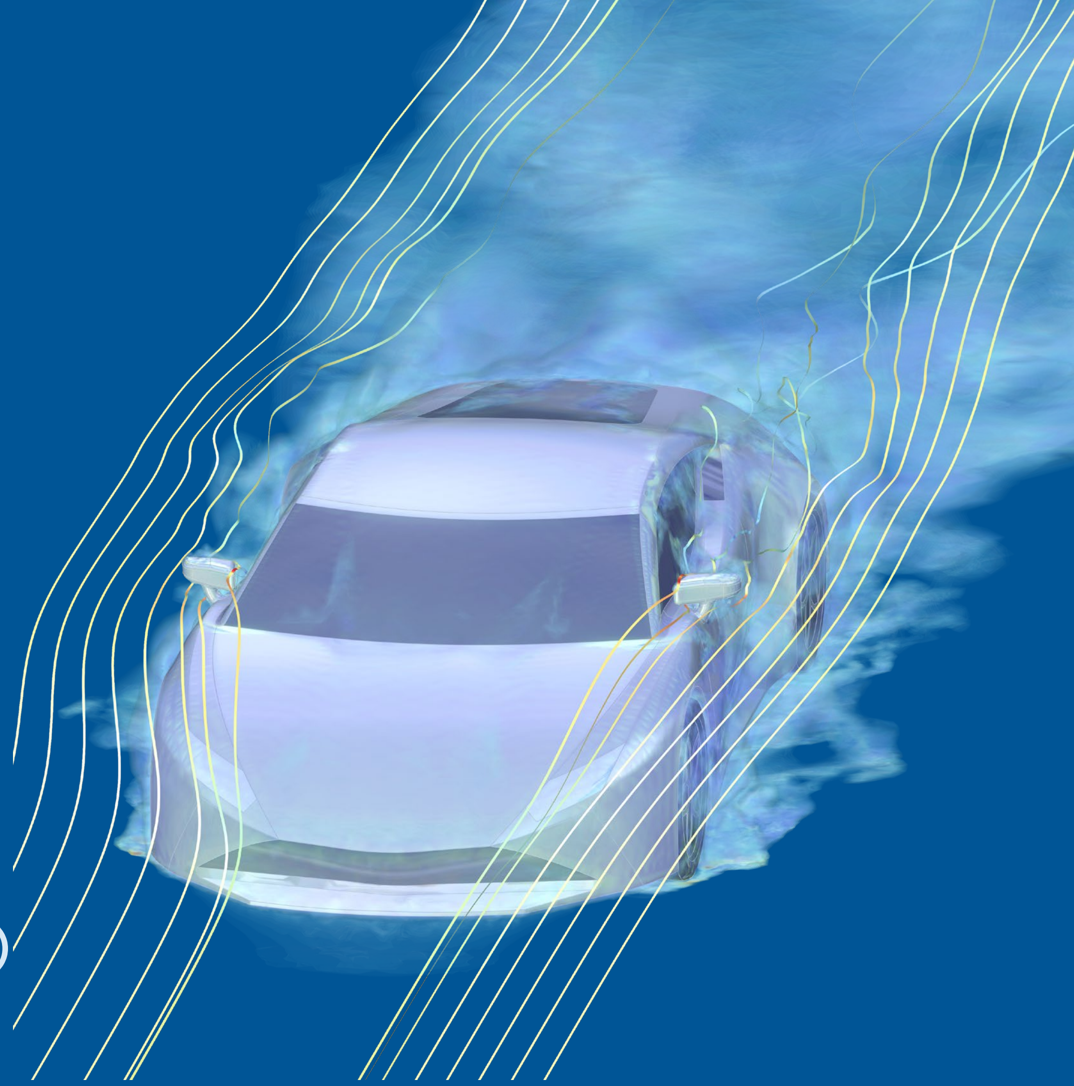


COMSOL Multiphysics®
Turbulent Flow Modeling

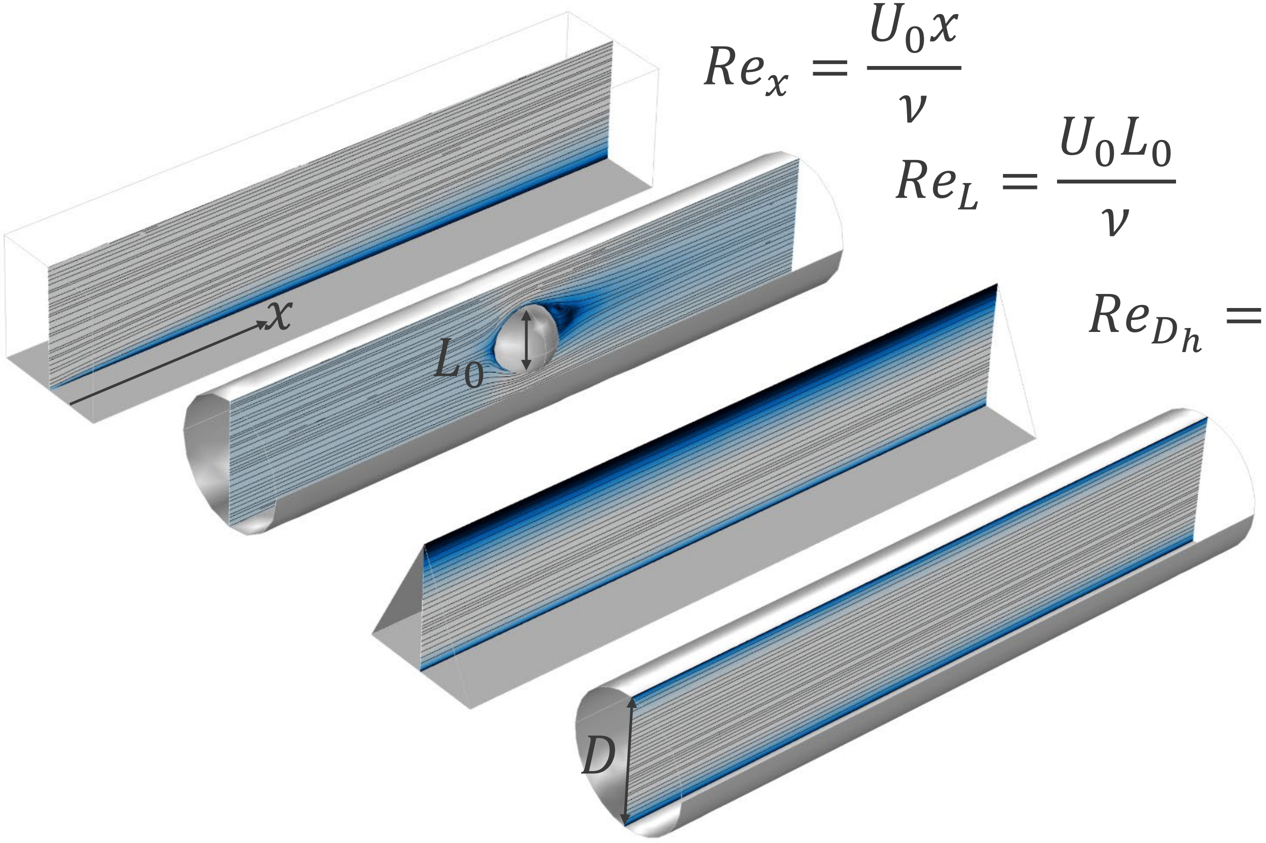
Turbulent Flow

Determining the Flow Regime, Re

- At low Re ($\ll 1$): creeping (Stokes) flow
  - Viscous forces tend to damp out all flow disturbances
  - Reversible smooth flow pattern
- At intermediate Re ($\sim 1 - 2000^*$): laminar flow
  - Inertial forces become increasingly important
  - Viscous forces are confined to boundary layers, shear layers, and wakes
  - Regular, smooth flow pattern
- At high Re ($> 4000^*$): turbulent flow
  - Flow disturbances grow by nonlinear interactions producing a cascade of eddies (vortices)
  - Viscous damping is active everywhere but only has a significant effect on the smallest eddies
  - Disordered (chaotic) flow pattern

**Critical Re of 2000 has been mentioned above as an example considering an internal channel flow*

Reynolds Number Examples



$$Re_x = \frac{U_0 x}{\nu}$$

$$Re_L = \frac{U_0 L_0}{\nu}$$

$$Re_{D_h} = \frac{U_0 D_H}{\nu}, D_H \equiv \frac{4A}{P}$$

$$Re_D = \frac{U_0 D}{\nu}$$

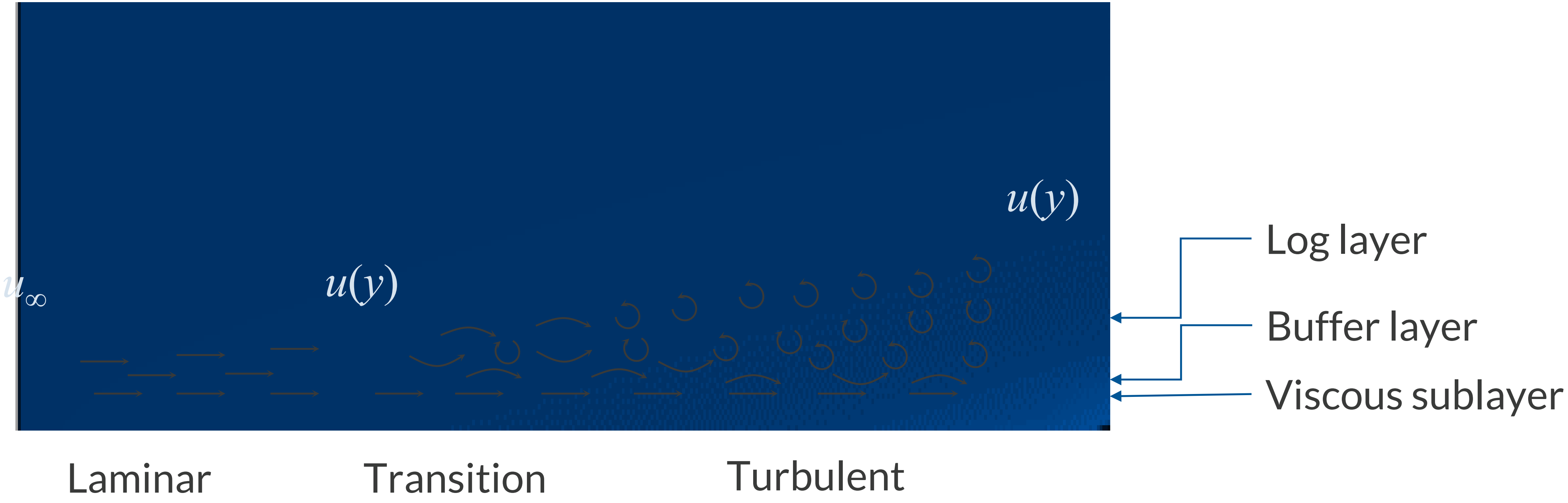
Determining the Flow Regime, Ma

- For $Ma = 0$: formally incompressible flow
  - Speed of sound is infinite (hypothetical case), instantaneous spread of pressure disturbances
- For $0 < Ma < 0.3$: weakly compressible flow
  - Density changes due to pressure by max. 5%
  - Density changes can also occur due to the dependency on temperature
- For $Ma > 0.3$: compressible flow
  - Thermodynamic effects not negligible
  - Transonic, supersonic flow with shock waves

Choosing the Right Fluid Flow Interface

- Fluid Flow
  - Single-Phase Flow — $\boldsymbol{Ma < 0.3}$
    - Creeping Flow (spf) — $Re \ll 1$
    - Laminar Flow (spf) — $Re \sim 1 - Re_{cr}$
    - Turbulent Flow — $Re > Re_{cr}$
    - Large Eddy Simulation — $Re > Re_{cr}$
    - Detached Eddy Simulation — $Re > Re_{cr}$
    - Rotating Machinery, Fluid Flow
    - Pipe Flow (pfl)
    - Viscoelastic Flow (vef)
    - Water Hammer (whtd)
  - Multiphase Flow
  - Porous Media and Subsurface Flow
  - Nonisothermal Flow
  - High Mach Number Flow — $\boldsymbol{Ma > 0.3}$
    - High Mach Number Flow, Laminar (hmnf) — $Re < Re_{cr}$
    - Turbulent Flow — $Re > Re_{cr}$
    - Compressible Euler Equations (cee) — $Re \rightarrow \infty$
    - Rotating Machinery, High Mach Number Flow
  - Particle Tracing
  - Fluid-Structure Interaction
  - Rarefied Flow
  - Thin-Film Flow

Laminar, Transitional, and Turbulent Flow



Introduce turbulent viscosity term (μ_T)

- Reynolds Averaged Navier-Stokes (RANS) equations:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla \cdot \overline{(\rho \mathbf{u}' \otimes \mathbf{u}')} = -\nabla P + \mu \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) + \mathbf{F}$$

$$\rho(\nabla \cdot \mathbf{u}) = 0$$

- Model the turbulent fluctuations using turbulent viscosity μ_T :

$$-\mu_T \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right)$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P + (\mu + \mu_T) \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) + \mathbf{F}$$

$$\rho(\nabla \cdot \mathbf{u}) = 0$$

In COMSOL: 9 different turbulence models to solve for μ_T = spf.muT

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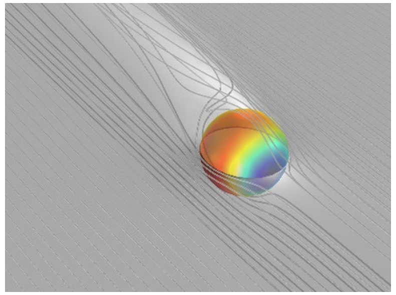
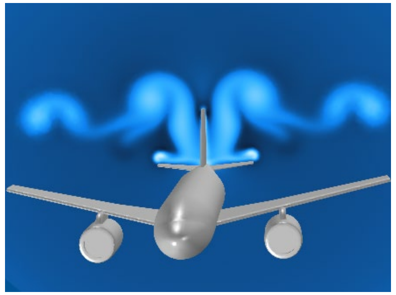
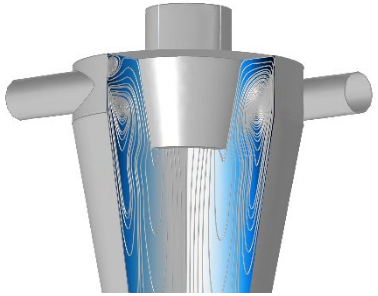
Computational Cost for Turbulence Models

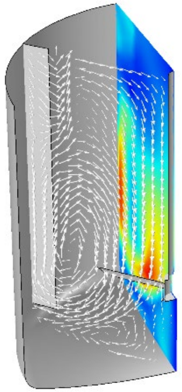
- Algebraic yPlus and L-VEL
- Spalart–Allmaras
- k-epsilon
- k-omega, SST and low Re k-epsilon
- Realizable k-epsilon
- v^2-f

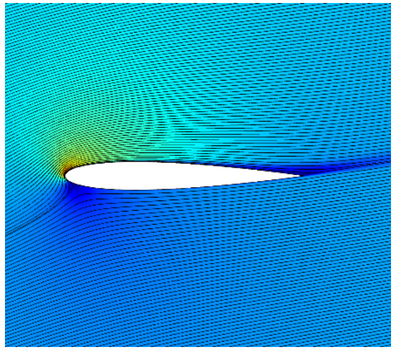
Increasing cost

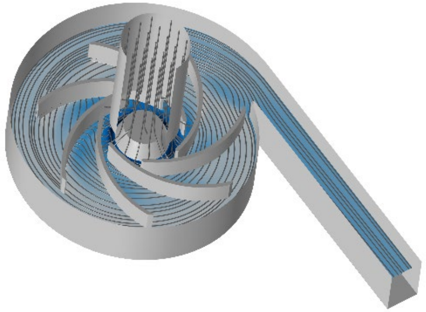












Algebraic Turbulence Models

- Algebraic turbulence models are faster and more robust but, generally less accurate than transport-equation turbulence models
- Reynolds number based on local velocity magnitude and wall distance

$$Re = \frac{\bar{\rho} U y}{\bar{\mu}} = \frac{U}{u_\tau} \frac{\bar{\rho} u_\tau y}{\bar{\mu}} = u^+ y^+, \qquad \bar{\mu} + \mu_T = \frac{\bar{\mu}}{\partial u^+ / \partial y^+}$$

- Algebraic yPlus model prescribes $u^+(y^+)$ from an extension of the logarithmic wall law and solves for y^+
- L-VEL model prescribes $y^+(u^+)$ from a different extension of the logarithmic wall law and solves for u^+
  - NOTE: No turbulent transport → no wake interference and no in/outflow of turbulence

Transport-Equation Turbulence Models (2-Eqn.)

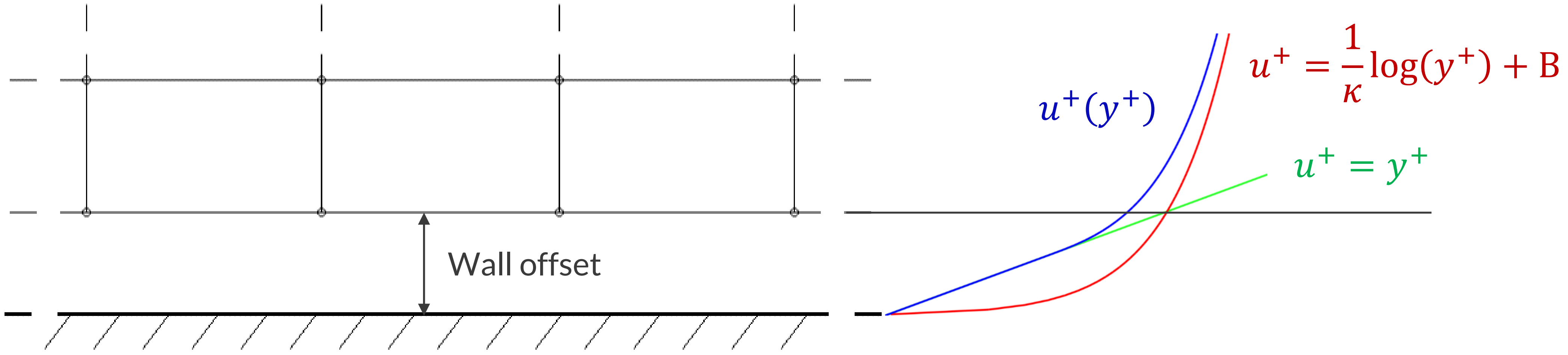
Transport-equation turbulence models solve for the transport of turbulence quantities.

- k- ε : Transport equations for k and ε. General purpose model with wall functions for smooth and rough walls.
- Realizable k- ε : Realizability constraints are built into the model parameters.
- k- ω : Transport equations for k and ω. More accurate than the k-epsilon model close to walls and in recirculation regions. Wall functions for smooth and rough walls but can also resolve the flow close to walls.
- SST: Combines the k- ω model close to walls with the k- ε model in the outer flow region. Can resolve the wall layer.
- Low Re k- ε : Wall resolved k- ε model.

Transport-Equation Turbulence Models (1 and 3 Eqn.)

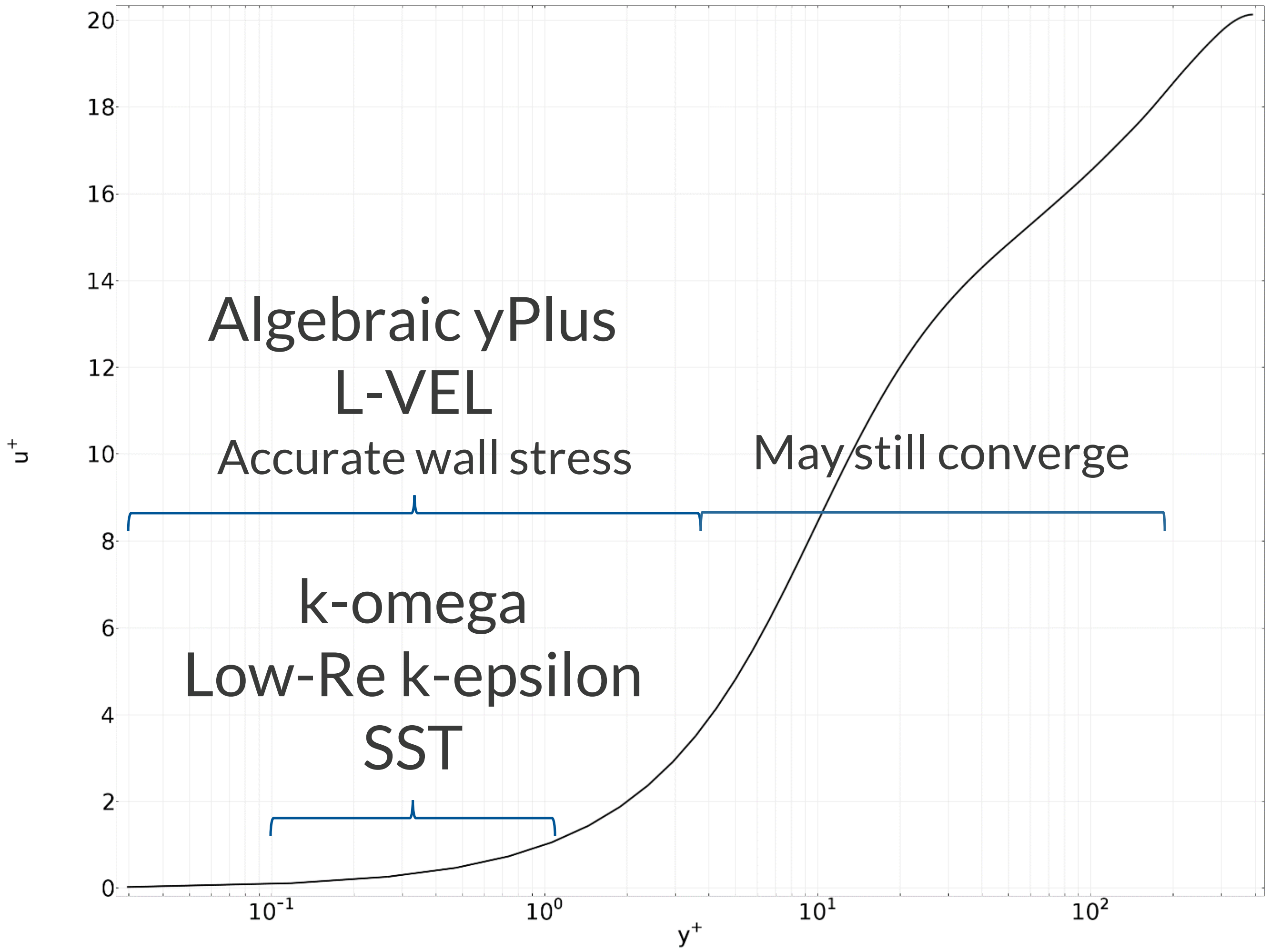
- Spalart–Allmaras: Transport equation for $\tilde{\upsilon}_T$. Wall resolved turbulence model developed for aerodynamic applications.
- v^2-f: An extended low-Re k-ε model. In addition to k and ε, it also solves for the wall normal fluctuations (v^2) and an elliptic relaxation function (f). Good predictive capabilities for flow over curved surfaces. Can resolve the wall layer.

Wall Treatment



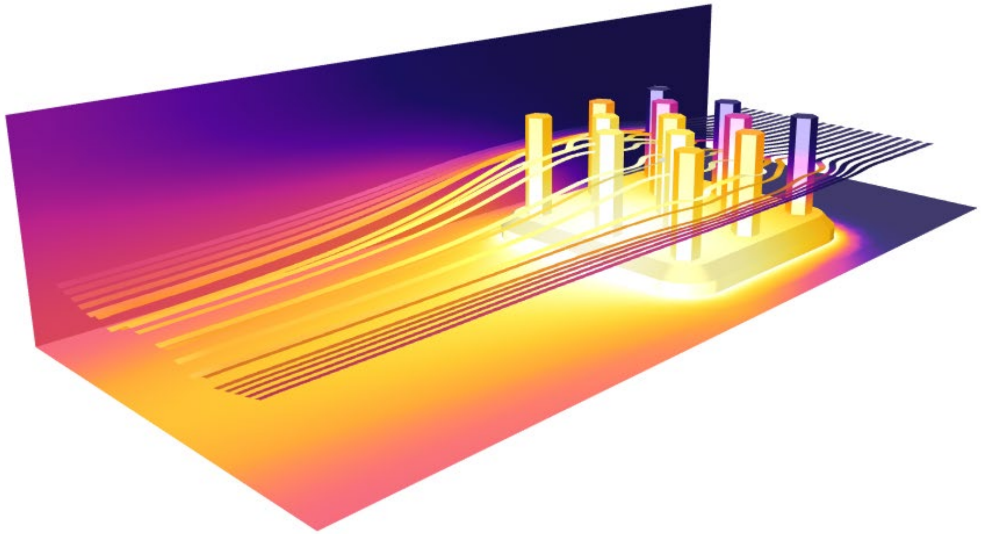
- Wall functions: A wall offset is applied. The computational domain always start in the log-layer.
- Low-Reynolds-number treatment: The viscous sublayer must be resolved.
- Automatic wall treatment: A wall offset is applied. The computational domain can start anywhere in the boundary layer.

Resolution for Low-Re Wall Treatment



Algebraic Turbulence Models

- Turbulent viscosity evaluated from the Reynolds number based on local speed and wall distance
  - Algebraic yPlus
  - L-VEL
- Advantages:
  - Robust
  - Computationally inexpensive
- Disadvantage:
  - Less accurate



Surface temperature and streamlines in a benchmark for electronic cooling.

Transport Equations for Turbulence: Two Equation Models

k-ε models

- The standard k-ε model with realizability constraints
- The realizable k-ε model

k-ω model

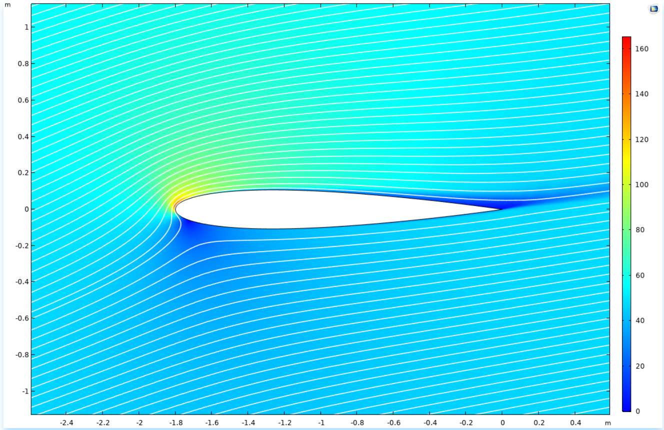
- The revised Wilcox k-ω model (1998) with realizability constraints

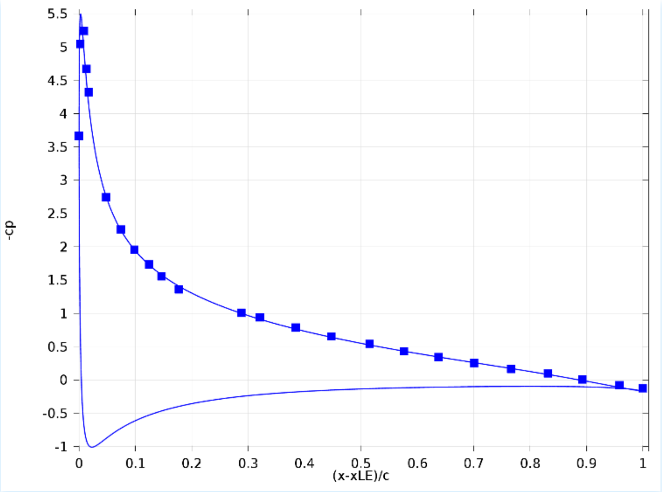
SST model

- Combines the k-ε model in the free stream with the k-ω model close to walls

Low Re k-ε model

- The AKN k-ε model





Benchmark model of a NACA0012 airfoil using the SST turbulence model.

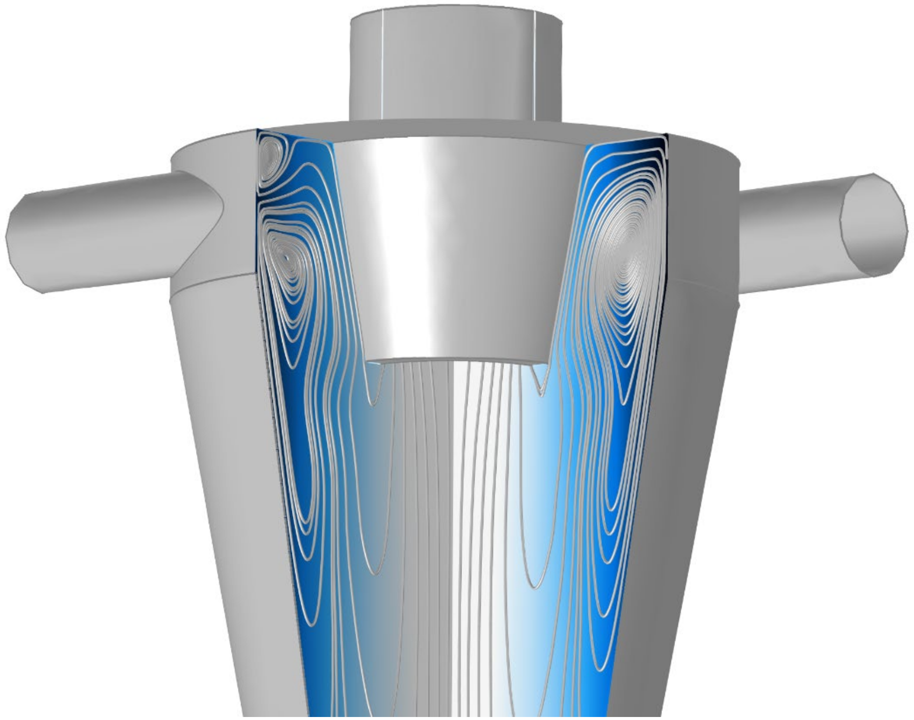
Transport Equation Turbulence Models

Spalart-Allmaras

- One-equation model with rotational correction, developed for aerodynamic applications

v^2-f model

- An extension of the k-ε model which accounts for turbulence anisotropy by solving for the wall-normal turbulence velocity fluctuations



Flow in a hydrocylone. A typical application where v^2-f gives superior results over two-equation models such as k-ε or SST.

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Wall Treatment

Wall functions for smooth and rough walls

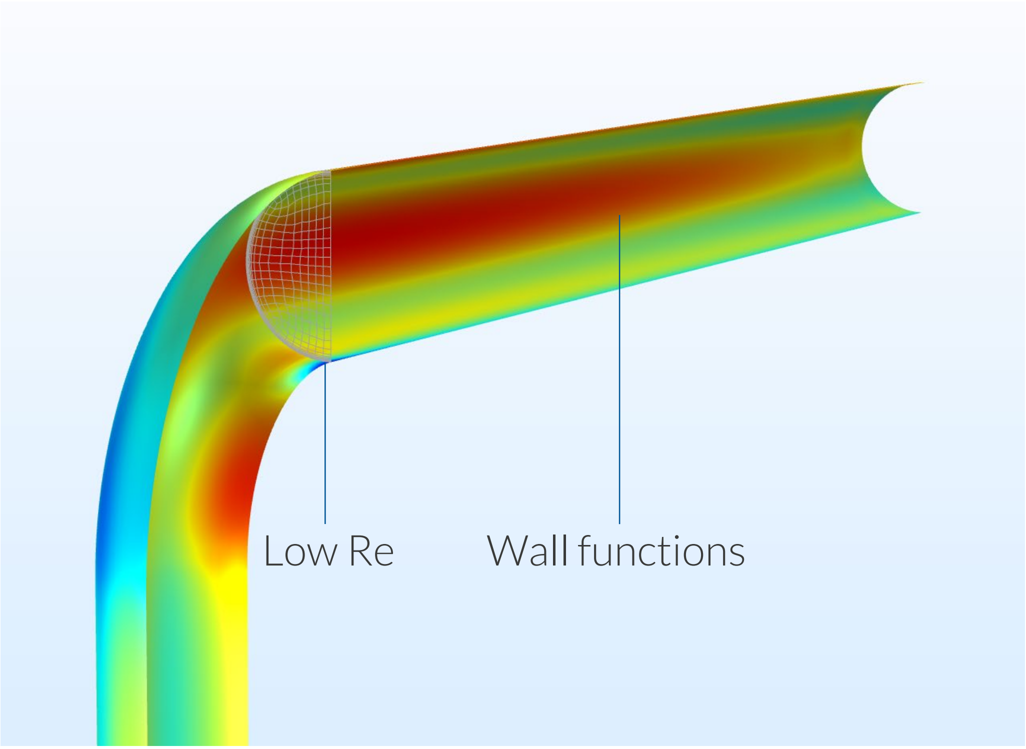
- Robust, applicable for coarse meshes, but limited accuracy
- Supported by k-ε, Realizable k-ε and k-ω

Low-Reynolds-number treatment

- Resolves the flow all the way down to walls, accurate
- Supported by all turbulence models except k-ε and realizable k-ε

Automatic wall treatment

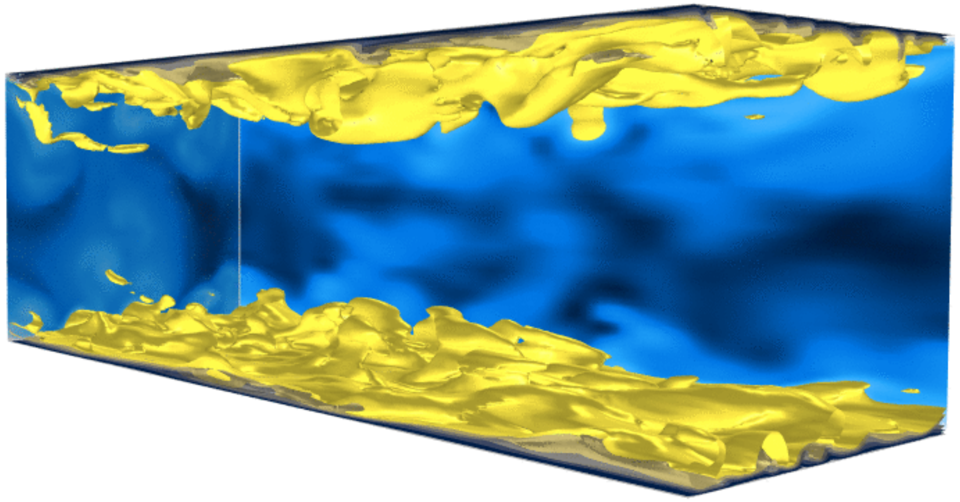
- Switches between low-Re treatment and wall functions, accuracy according to local mesh resolution
- Inherits the robustness provided by wall functions
- Default for all turbulence models except k-ε and realizable k-ε



Flow in a pipe elbow simulated with the k-ω model.

Large Eddy Simulation

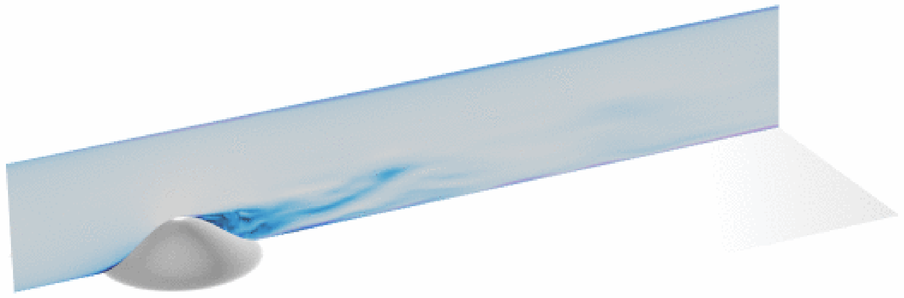
- The larger three-dimensional, unsteady eddies are resolved, whereas the effect of the smaller eddies is modeled.
- Simulations must be three-dimensional and time dependent.
- Computationally demanding
- The three current interfaces are based on variational multiscale methods
  - RBVM
  - RBVMWV
  - Smagorinsky



LES of turbulent channel flow at $Re_T = 395$

Detached Eddy Simulation

- Hybrid method between RANS and large eddy simulation (LES)Simulations must be three-dimensional and time dependent.
- RANS is used in the boundary layer and LES is used elsewhere.
- Requires a less dense boundary layer mesh compared with a pure LES which substantially reduces the memory requirement and computation
- Spalart–Allmaras turbulence model with the following LES models
  - RBVM
  - RBVMWV
  - Smagorinsky

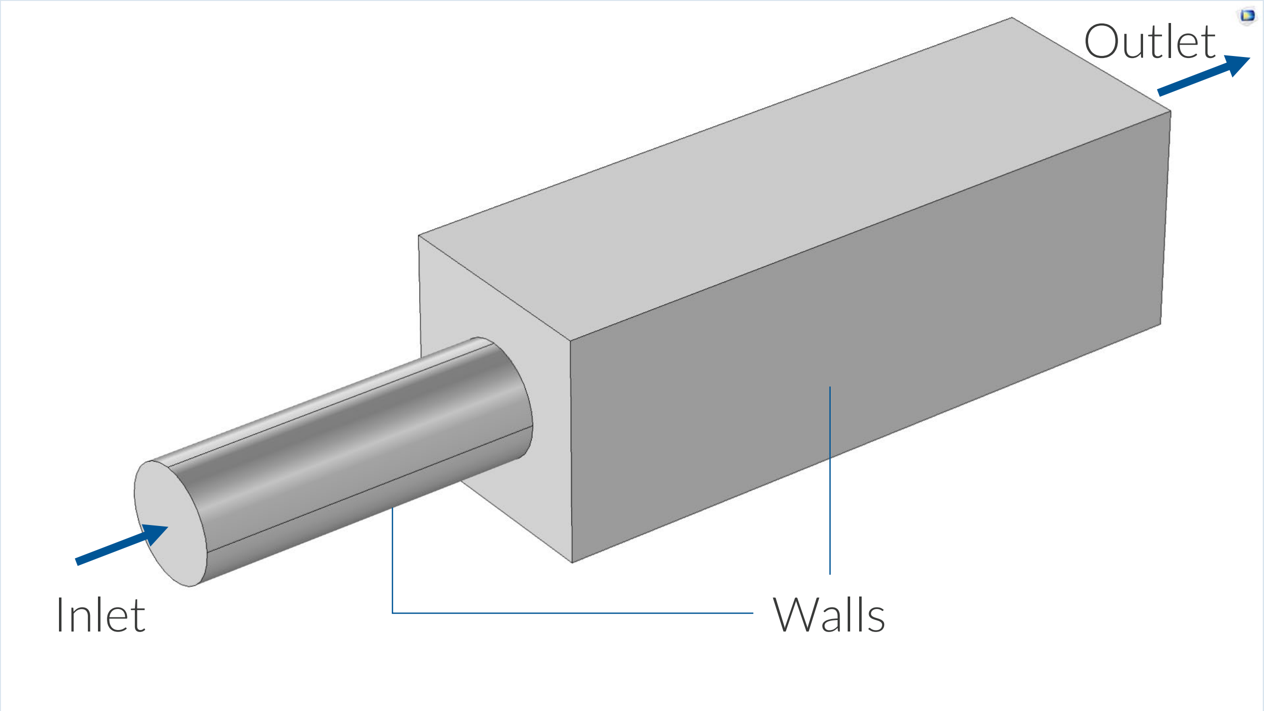


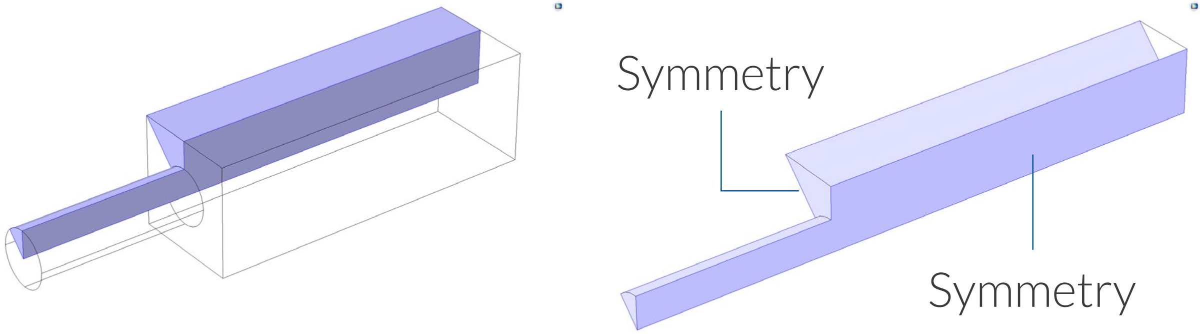
Flow over an Obstacle with DES

Demo

Model Definition

- Laminar flow in water
- Fully developed flow at the inlet
- Pressure condition at the outlet
- No-slip conditions at walls
- Symmetry conditions at the two lateral surfaces
- Why?
  - Typical expansion found in many systems, e.g. in medtech
  - Benchmark with flow separation





Due to symmetry, we only have to model one eight of the model domain, provided that the flow is steady and that the inlet flow is perpendicular to the inlet boundary.

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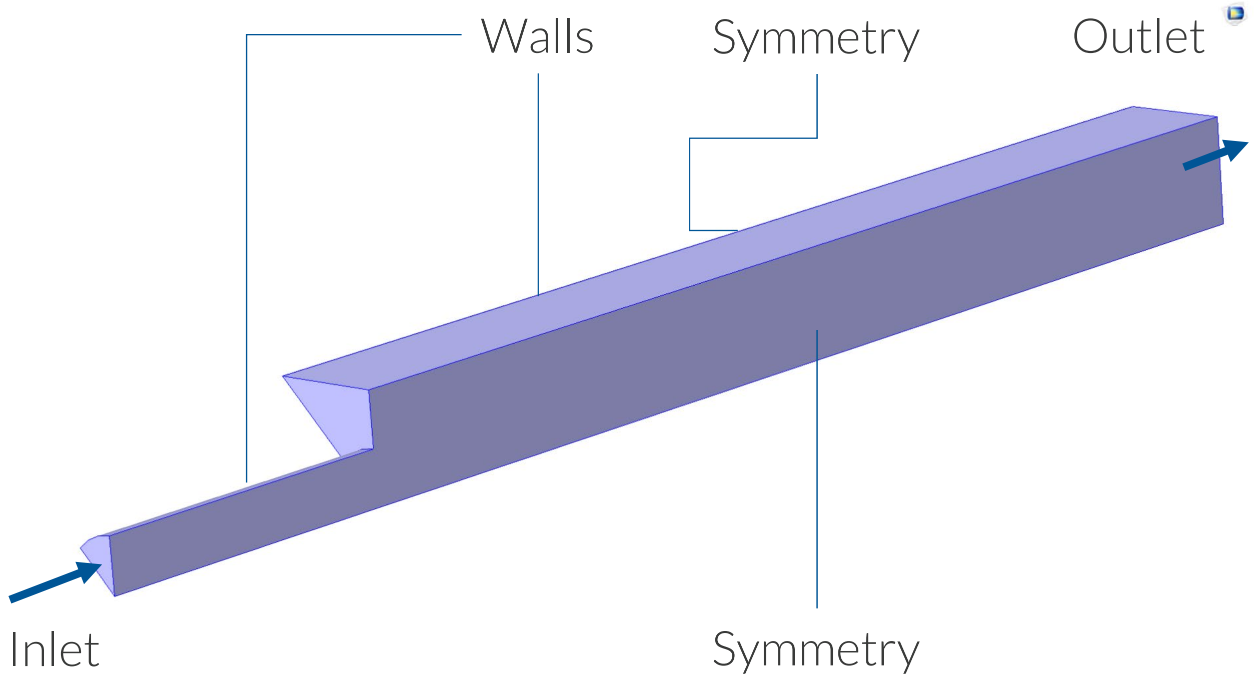
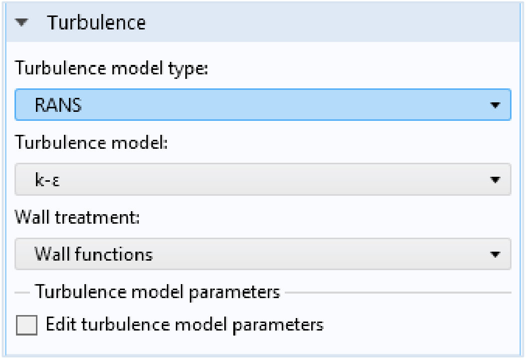
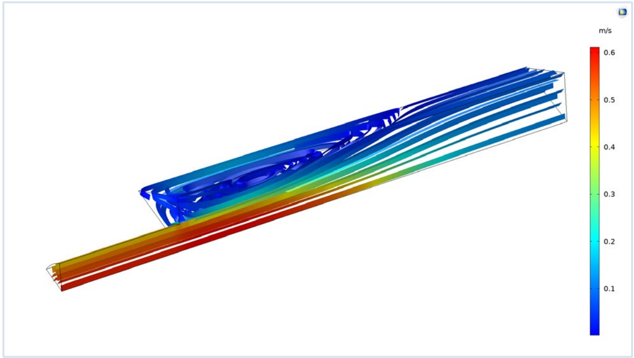
Model Definition

- Extension of previous laminar flow example:
  - Elongate outlet section to avoid recirculation zone close to outlet
- Turbulent flow in water
  - k-ε turbulence model
- Fully developed flow at the inlet
- Pressure condition at the outlet
- Wall functions at walls
- Symmetry conditions at the two lateral surfaces







It is possible to change the model settings from laminar flow to turbulent flow and also chose turbulence model.

Results

- Flow and pressure fields
- Length of recirculation zone
- Note:
  - Recirculation reaches the outlet
    -> elongate the outlet section