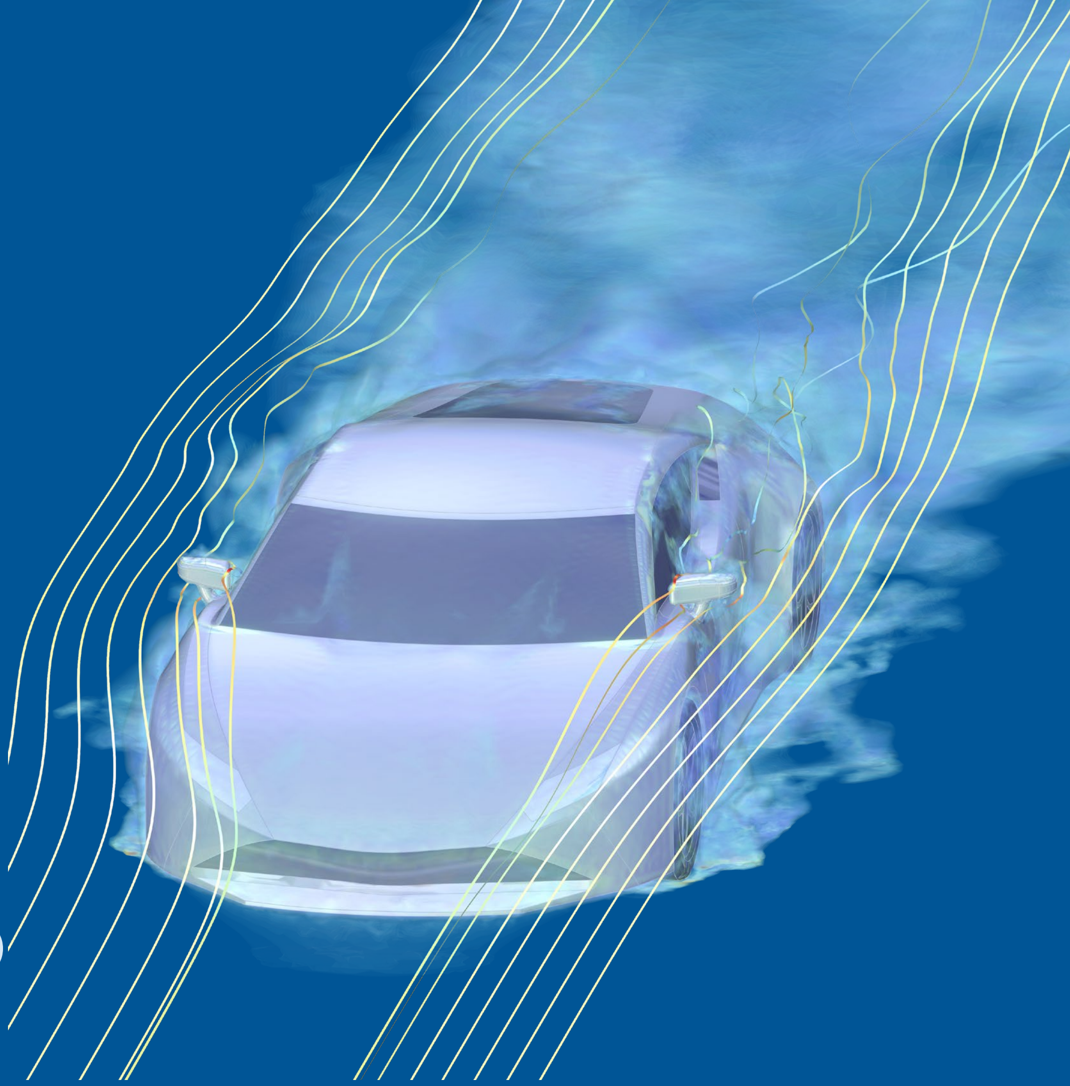

COMSOL Multiphysics®
Laminar Flow Modeling

Laminar Flow

Single-Phase Flow Equations

- Navier–Stokes, continuity, and energy equation,

$$\rho \frac{D\boldsymbol{u}}{Dt} = -\boldsymbol{\nabla} p + \boldsymbol{\nabla} \cdot (2\mu \boldsymbol{S}) + \boldsymbol{F}$$

$$\frac{D\rho}{Dt} + \rho \boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

$$\rho C_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + 2\mu \boldsymbol{S} : \boldsymbol{S} + \alpha_p T \frac{Dp}{Dt} + Q$$

- where,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla, \qquad \boldsymbol{S} = \frac{1}{2}\left(\boldsymbol{\nabla} \boldsymbol{u} + (\boldsymbol{\nabla} \boldsymbol{u})^{\mathrm{T}} - \frac{2}{3} (\boldsymbol{\nabla} \cdot \boldsymbol{u}) \mathbf{I} \right)$$

Special Case: Perfect Gas

- For a perfect gas,

$$p = \rho R T$$

- Using the energy equation, the continuity equation may be recast as,

$$\frac{1}{\gamma p}\frac{Dp}{Dt} + \frac{\nabla \cdot (k \nabla T)}{\rho C_p T} + \frac{2\mu \boldsymbol{S}:\boldsymbol{S}}{\rho C_p T} + \frac{Q}{\rho C_p T} + \nabla \cdot \boldsymbol{u} = 0$$

- Introducing scales for the dependent variables, U_0, T_0, p_0, properties, ρ_0, μ_0, k_0, and L_0 (length), and L_0/U_0 (time). Scale pressure changes with dynamic pressure, $\Delta p \sim \rho_0 U_0^2$,

$$\frac{1}{\gamma p}\frac{Dp}{Dt} + \frac{\nabla \cdot (k \nabla T)}{\rho C_p T} + \frac{2\mu \boldsymbol{S}:\boldsymbol{S}}{\rho C_p T} + \frac{Q}{\rho C_p T} + \nabla \cdot \boldsymbol{u} = 0$$

$$\frac{U_0}{L_0}\left(\sim Ma^2 \quad \sim \frac{1}{Pe} \quad \sim \frac{(\gamma - 1) Ma^2}{Re} \quad \sim \frac{1}{Pe}\frac{Q L_0^2}{k_0 T_0} \quad \sim 1 \right) = 0$$

Dimensionless Numbers

- Similarly, for the momentum equation,

$$\rho \frac{D\boldsymbol{u}}{Dt} + \nabla p - \nabla \cdot (2\mu \boldsymbol{S}) - \boldsymbol{F} = 0$$

$$\frac{\rho_0 U_0^2}{L_0}\left(\sim 1 \qquad \sim 1 \qquad \sim \frac{1}{Re} \qquad \sim 1 \right) = 0$$

- The dimensionless numbers are,
  - Reynolds number (inertial forces/viscous forces): $Re = \frac{\rho_0 U_0 L_0}{\mu_0}$
  - Mach number (flow speed/speed of sound in fluid): $Ma = \frac{U_0}{a_0}, \quad a_0 = \frac{\gamma p_0}{\rho_0}$
  - Peclet number (convection/conduction): $Pe = \frac{\rho_0 C_p U_0 L_0}{k_0}$

Determining the Flow Regime, Re

- At low Re ($\ll 1$): creeping (Stokes) flow
  - Viscous forces tend to damp out all flow disturbances
  - Reversible smooth flow pattern
- At intermediate Re ($\sim 1 - 2000^*$): laminar flow
  - Inertial forces become increasingly important
  - Viscous forces are confined to boundary layers, shear layers, and wakes
  - Regular, smooth flow pattern
- At high Re ($> 4000^*$): turbulent flow
  - Flow disturbances grow by nonlinear interactions producing a cascade of eddies (vortices)
  - Viscous damping is active everywhere but only has a significant effect on the smallest eddies
  - Disordered (chaotic) flow pattern

**Critical Re of 2000 has been mentioned above as an example considering an internal channel flow*

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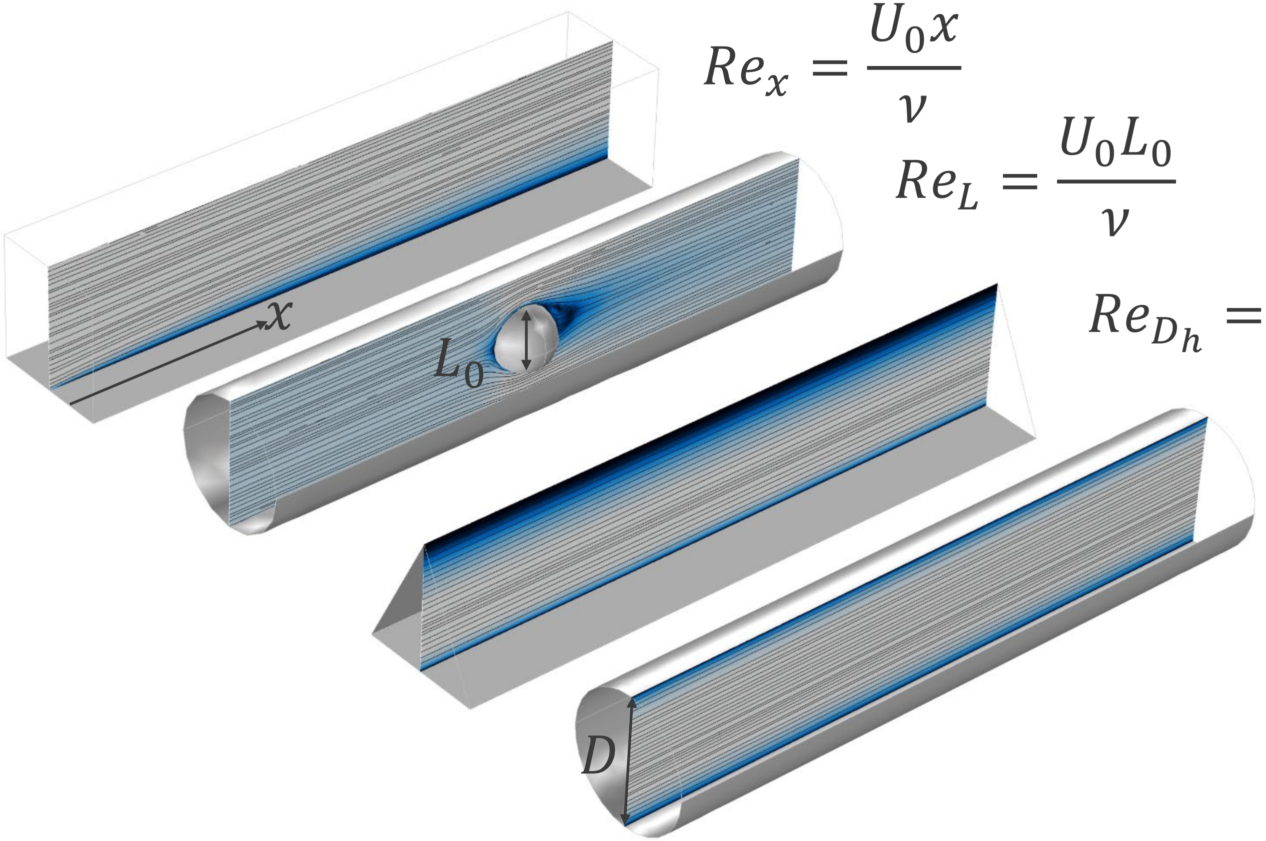
Reynolds Number Examples



$$Re_x = \frac{U_0 x}{\nu}$$

$$Re_L = \frac{U_0 L_0}{\nu}$$

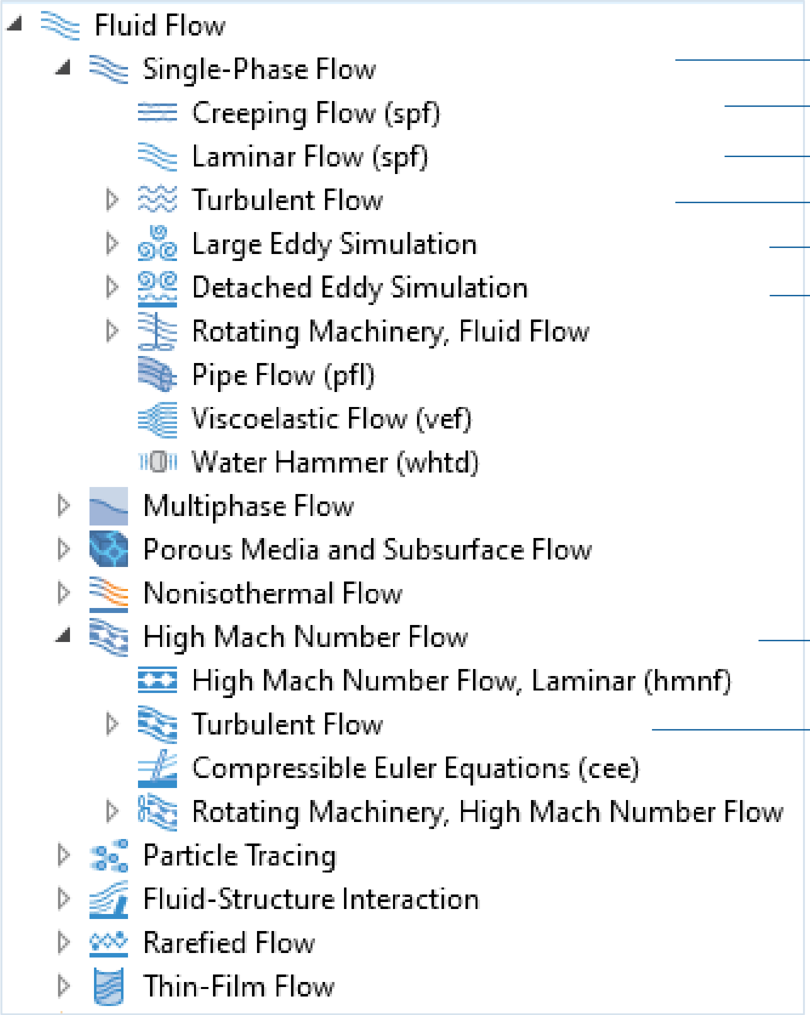
$$Re_{D_h} = \frac{U_0 D_H}{\nu}, D_H \equiv \frac{4A}{P}$$

$$Re_D = \frac{U_0 D}{\nu}$$

Determining the Flow Regime, Ma

- For $Ma = 0$: formally incompressible flow
  - Speed of sound is infinite (hypothetical case), instantaneous spread of pressure disturbances
- For $0 < Ma < 0.3$: weakly compressible flow
  - Density changes due to pressure by max. 5%
  - Density changes can also occur due to the dependency on temperature
- For $Ma > 0.3$: compressible flow
  - Thermodynamic effects not negligible
  - Transonic, supersonic flow with shock waves

Choosing the Right Fluid Flow Interface

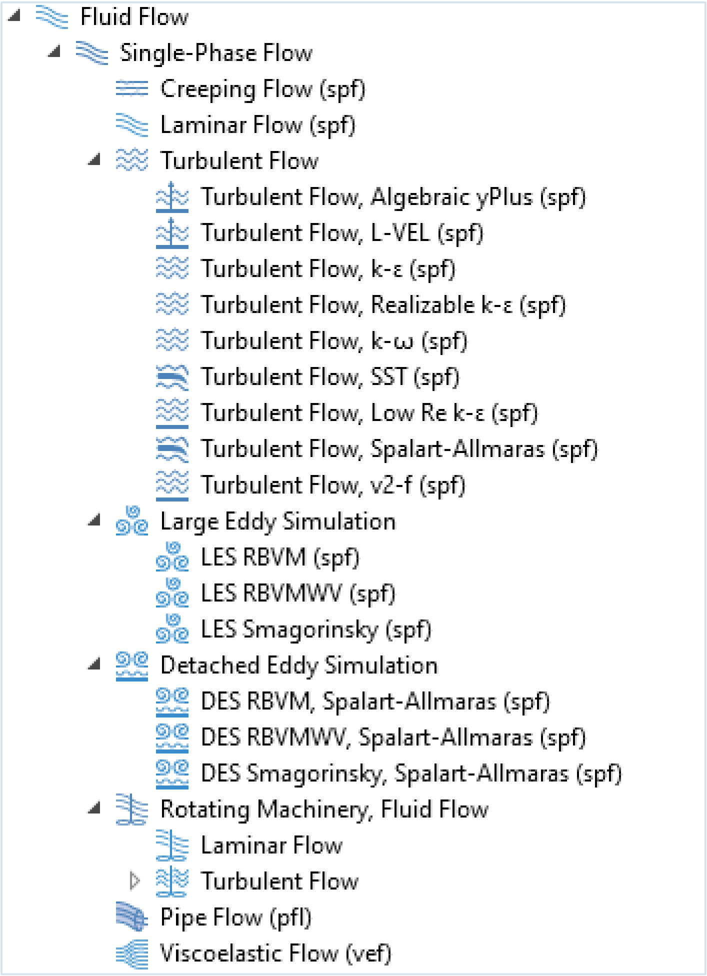


$\boldsymbol{Ma < 0.3}$
$Re \ll 1$
$Re \sim 1 - Re_{cr}$
$Re > Re_{cr}$
$Re > Re_{cr}$
$Re > Re_{cr}$

$\boldsymbol{Ma > 0.3}$
$Re < Re_{cr}$
$Re > Re_{cr}$
$Re \rightarrow \infty$

Single-Phase Flow

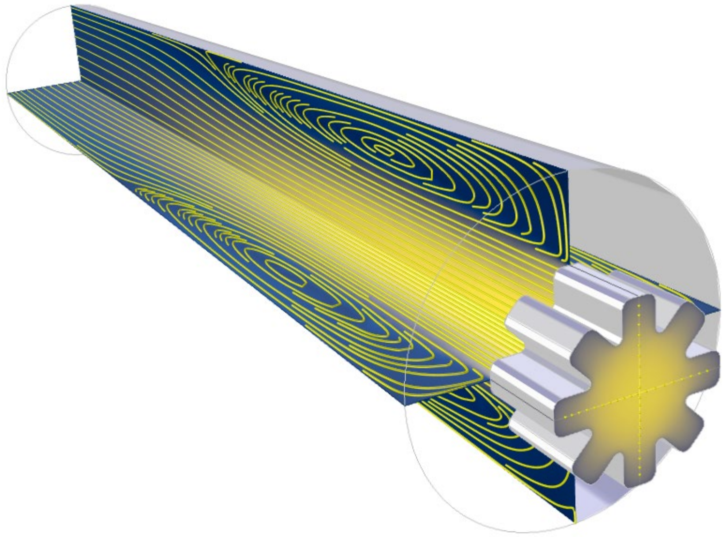
- Creeping flow also called Stokes flow
- Laminar flow
  - Newtonian and non-Newtonian flow
- Turbulent flow
  - 2 algebraic models
  - 7 transport-equation models
- Large eddy simulation (LES)
  - 3 variational multi-scale models
- Detached eddy simulation (DES)
- Rotating machinery
  - Laminar and turbulent flow
- Viscoelastic flow



The Single-Phase Flow user interfaces as displayed in the Physics list in the CFD Module.

Single-Phase Flow: General Functionality

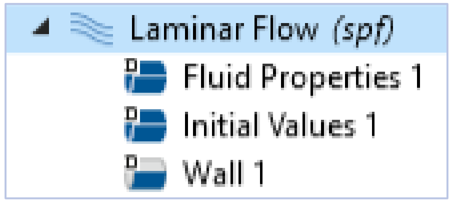
- Swirl flow
  - Includes the out-of-plane velocity component for axisymmetric flows
- Enable porous media domains
  - Model porous media flow or coupled free and porous media flow
- Gravity and reduced-pressure option
- Special purpose boundary conditions
  - Fully developed laminar and turbulent inflow and outflow
  - Wall conditions on internal shells for simulating thin immersed structures
  - Screen conditions for simulating thin perforated plates and wire gauzes

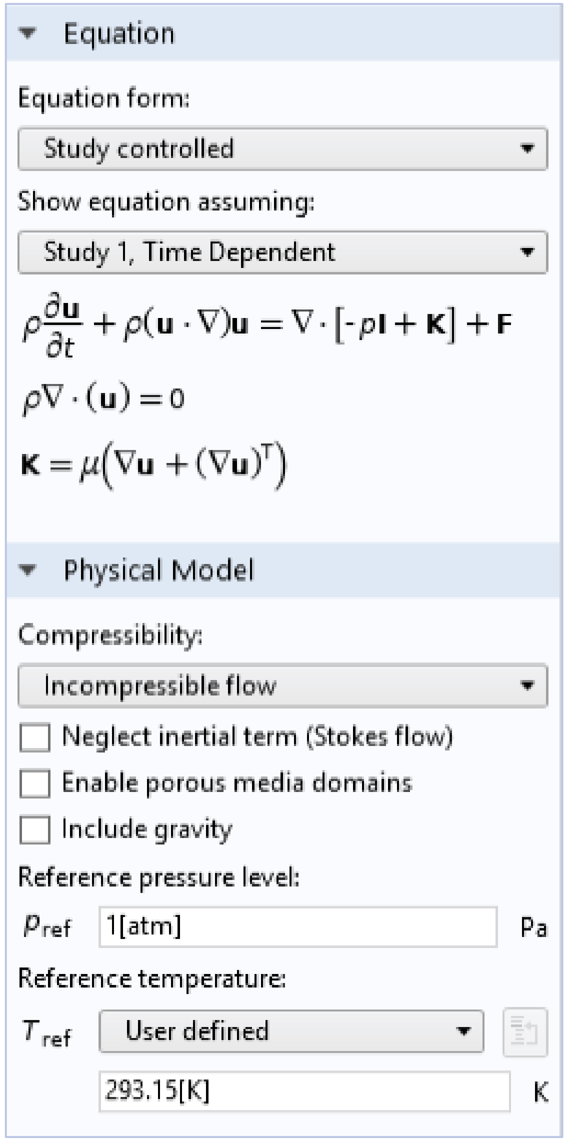


Fully developed turbulent flow condition at a star-shaped inlet.

Laminar Flow

- Solves the Navier–Stokes and continuity equations
  - Stationary or time dependent
  - Incompressible, weakly compressible, or compressible flow (Ma < 0.3)
- Neglect inertial term for Re << 1
- Enable porous media domains
- Include gravity
- Define reference pressure level and reference temperature



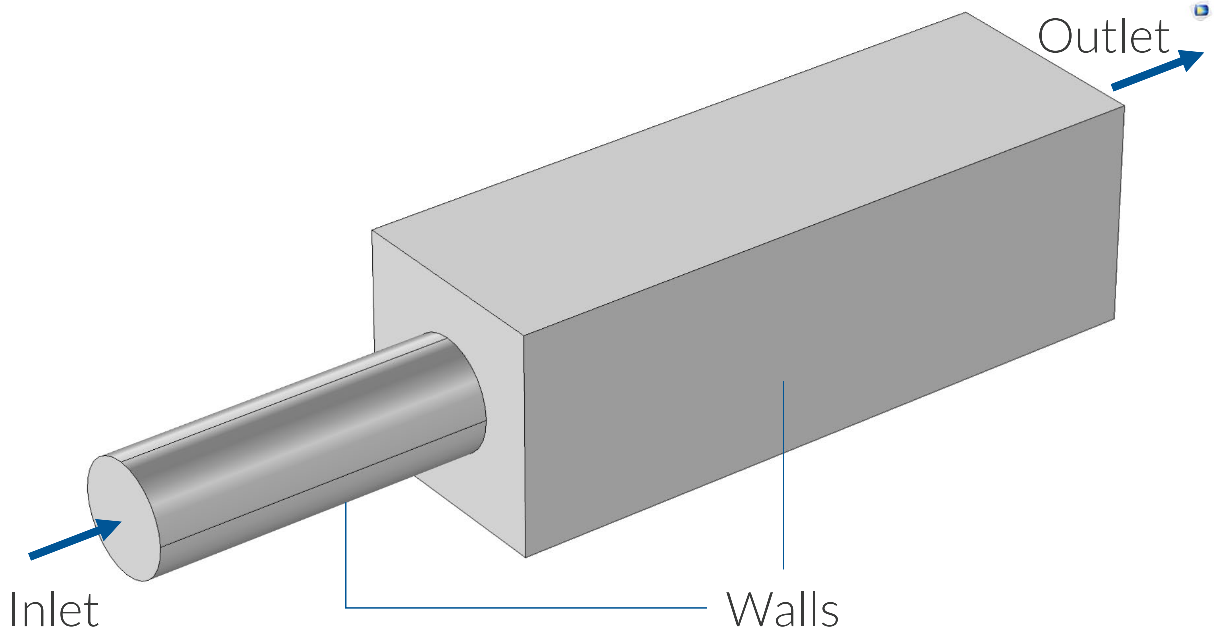


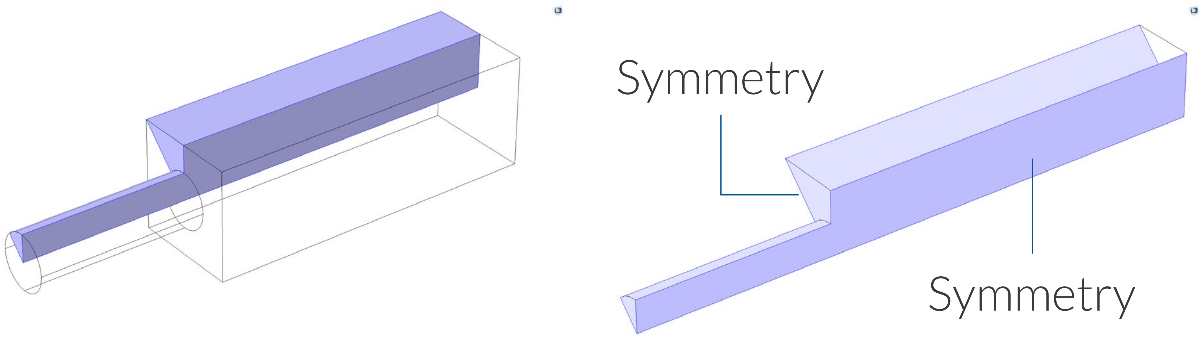
Laminar Flow node and Settings window

Demo

Model Definition

- Laminar flow in water
- Fully developed flow at the inlet
- Pressure condition at the outlet
- No-slip conditions at walls
- Symmetry conditions at the two lateral surfaces
- Why?
  - Typical expansion found in many systems, e.g. in medtech
  - Benchmark with flow separation

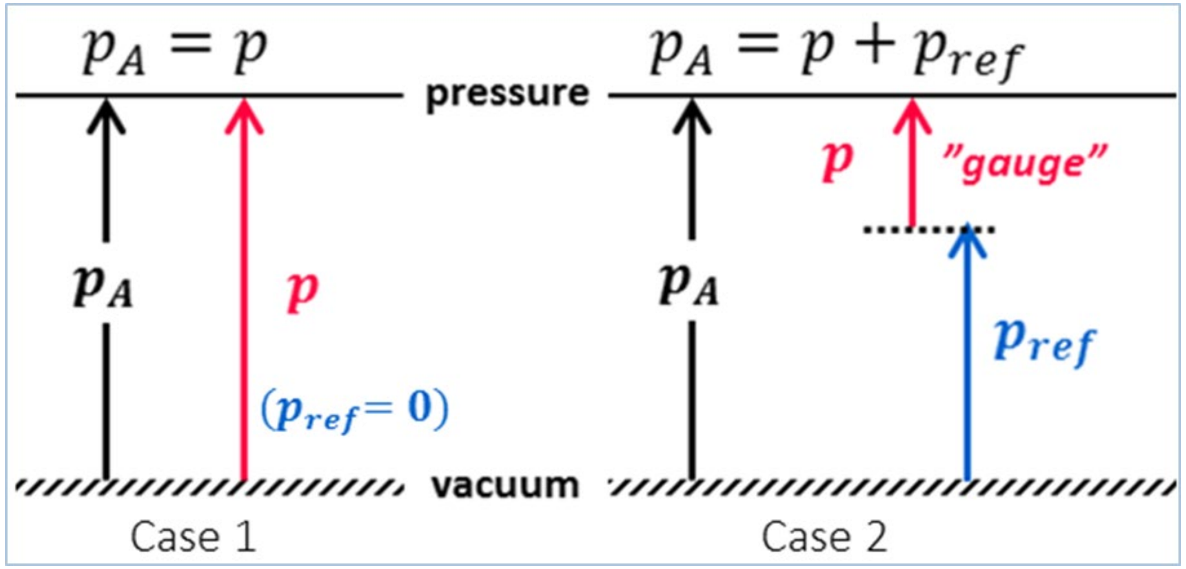




Due to symmetry, we only have to model one eight of the model domain, provided that the flow is steady and that the inlet flow is perpendicular to the inlet boundary.

About Fluid Pressure

- Flow is driven by pressure gradients
- Absolute pressure needed for material property evaluation; e.g., $\rho(p_A, T)$
- Generally two ways to calculate the pressure:
  - Case 1: $\Delta p \sim p_A$ → set $p_{ref} = 0$ and solve for the absolute pressure directly $p_A = p$
  - Case 2: $\Delta p \ll p_A$ → set p_{ref} close to typical system pressure level and solve for the (small) gauge pressure only $p_A = p_{ref} + p$
- Why is it important? → solver stability & convergence



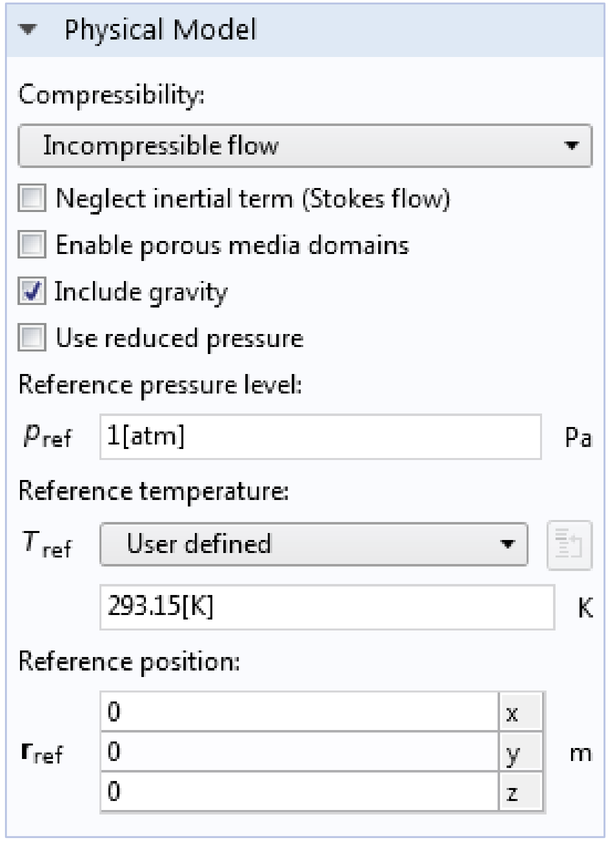
Gravity Property

- Includes a volume force on all domains,

$$\mathbf{F} = \rho \boldsymbol{g}$$

- Option to use reduced pressure for incompressible flow
- Boussinesq approximation available for nonisothermal flow

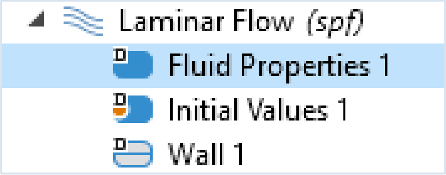
Section of the settings window in the fluid flow interface



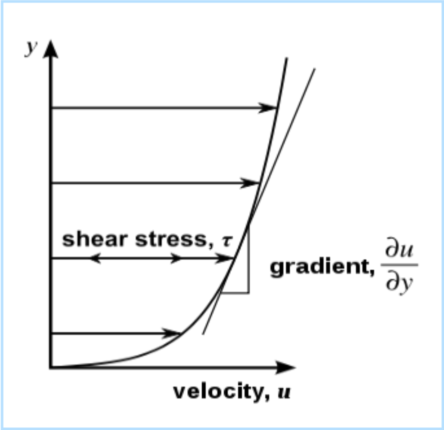
Lock-release gravity current

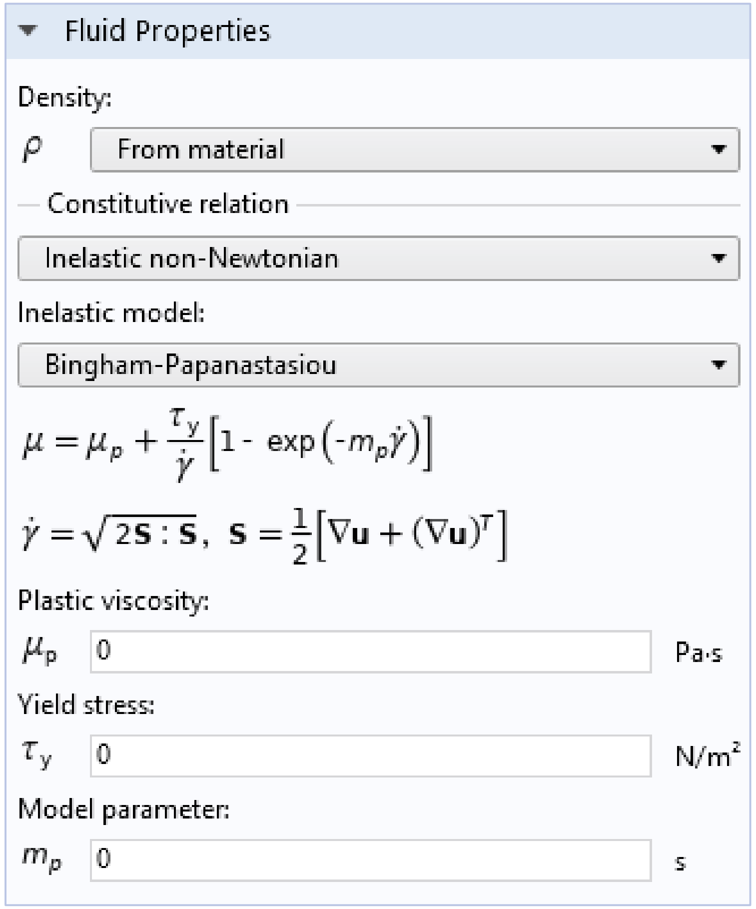
Fluid Properties

- Density
  - From material
  - User defined
- Constitutive relation
  - Newtonian
    - Dynamic viscosity
  - Inelastic non-Newtonian
    - Inelastic model



$$\tau = \mu\dot{\gamma} = \mu\frac{\partial u}{\partial y}$$

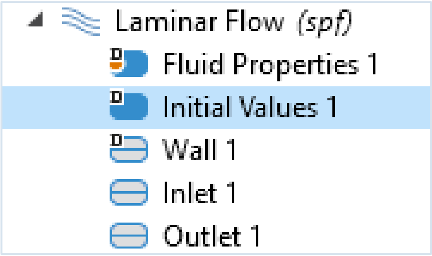


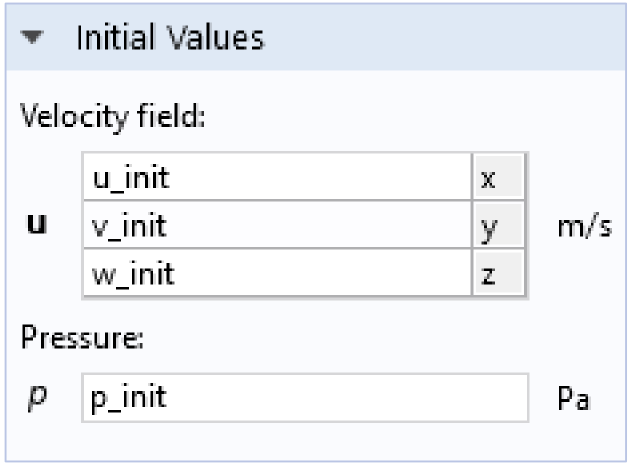


Newtonian and non-Newtonian constitutive relations

Initial Values

- Stationary
  - Initial values = initial guess for numerical solver
  - No physical meaning but starting point for iterations
  - Boundary conditions do not have to be consistent with initial values

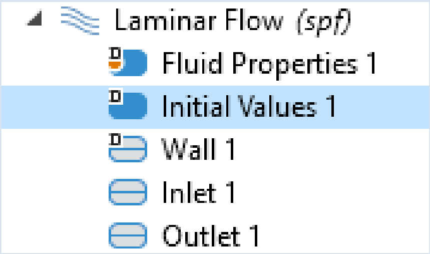


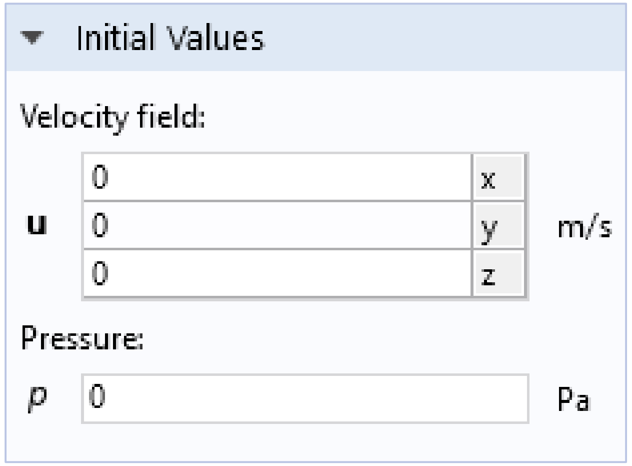


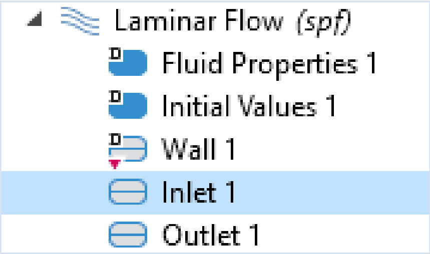
Initial Values *node and Settings window*

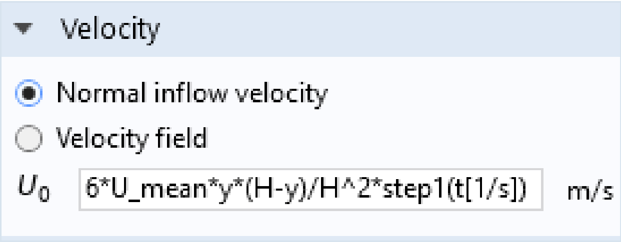
Initial Values

- Time dependent
  - Initial values = initial state of the flow and pressure fields
  - All boundary conditions must be consistent with initial condition (at t = 0)
    - Use a converged stationary solution as initial condition for the time-dependent study
    - Alternatively, use the trivial initial condition (***u***, *p* = 0) and ramp up the flow with time at the boundaries





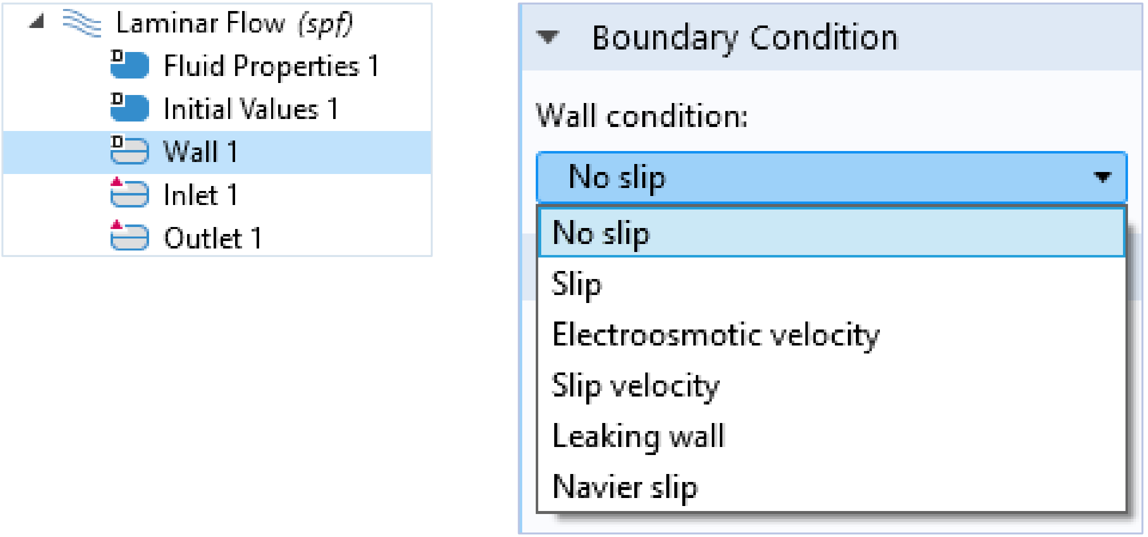




Trivial initial values *require that the flow is ramped up with time at the boundaries, here with a step function.*

Wall

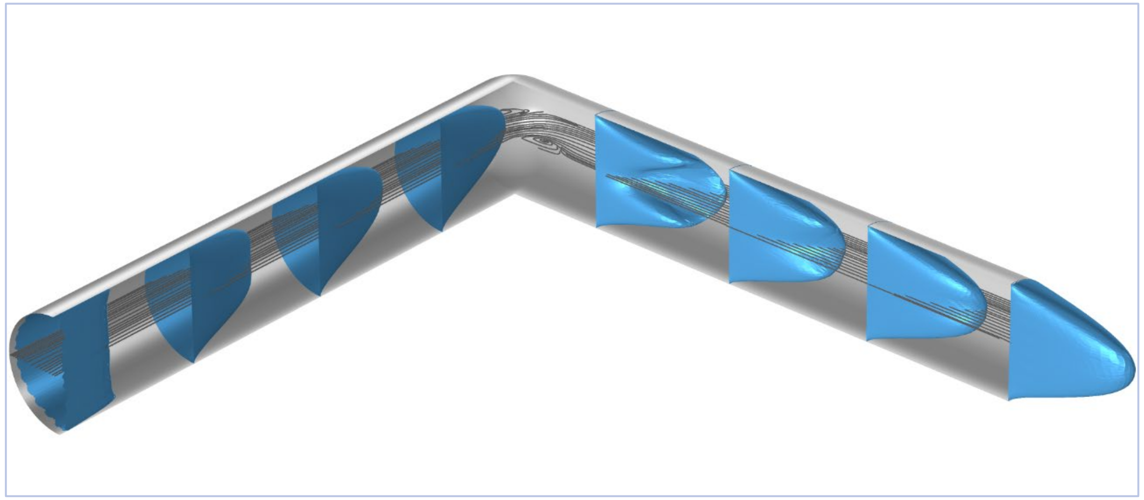
- Wall conditions:
  - No slip
  - Slip
  - Electroosmotic velocity
  - Slip velocity
  - Leaking wall
  - Navier slip
- Wall movement can be picked up automatically when a moving frame is defined in the model

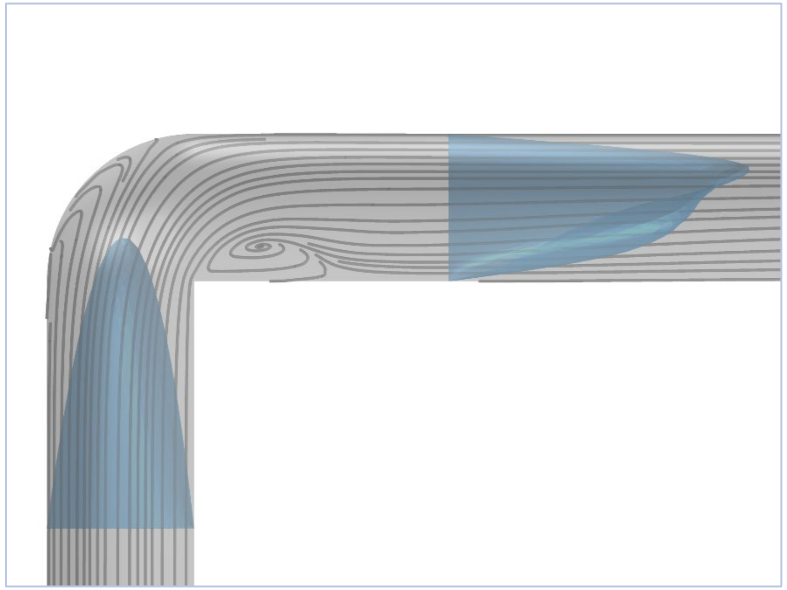


Wall node and Settings window

Location of Outlet Boundary

- Extend the domain:
  - Avoid recirculation and other complex flow patterns at the outlet



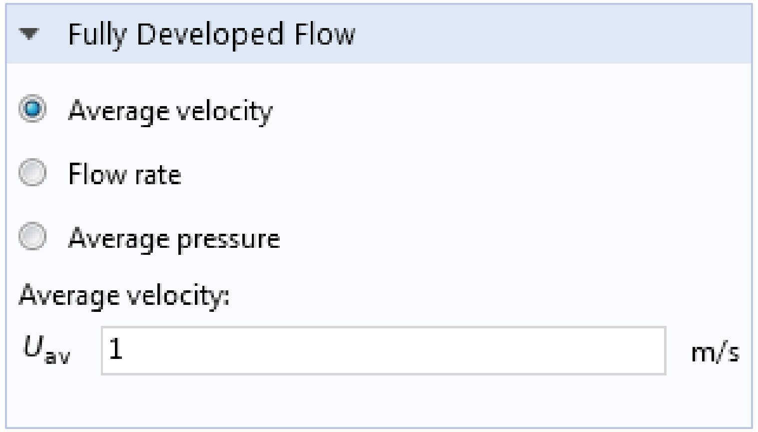


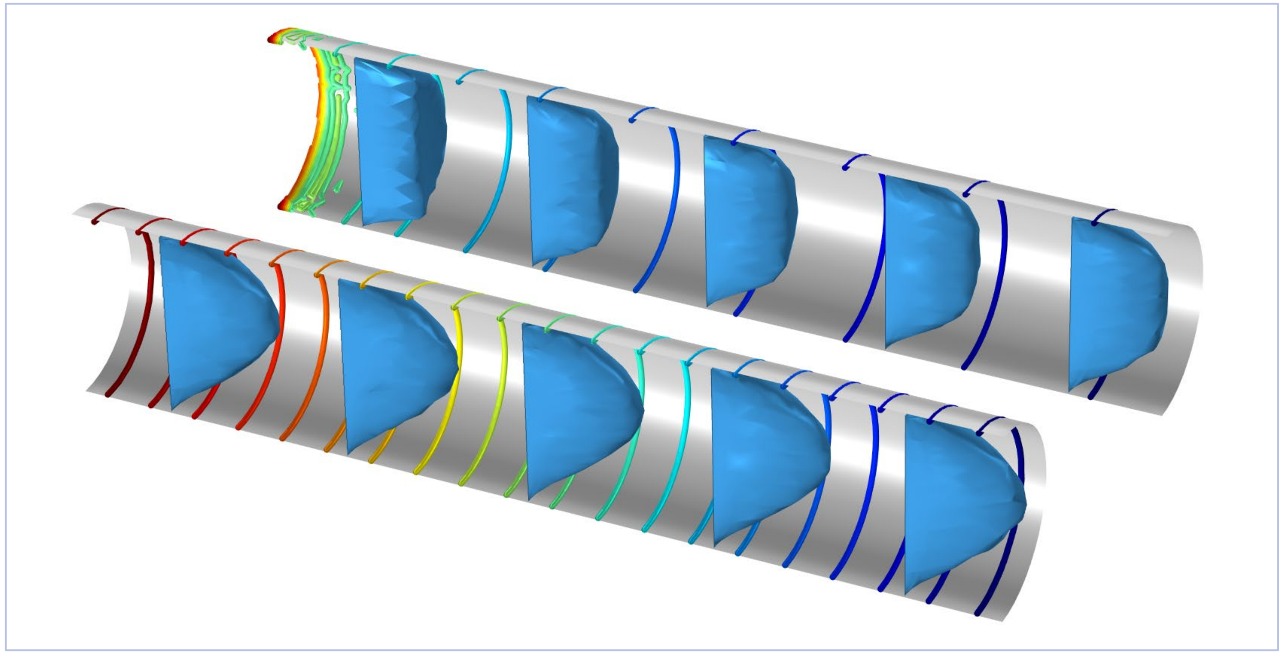
Flow in a semicircular pipe with a 90-degree bend

Fully Developed Flow

- Available for inlets and outlets
- Equations for fully developed flow are solved at the inlet or outlet boundary

$$L_E = 0.05 D_H \left(\frac{U D_H}{\nu} \right)$$

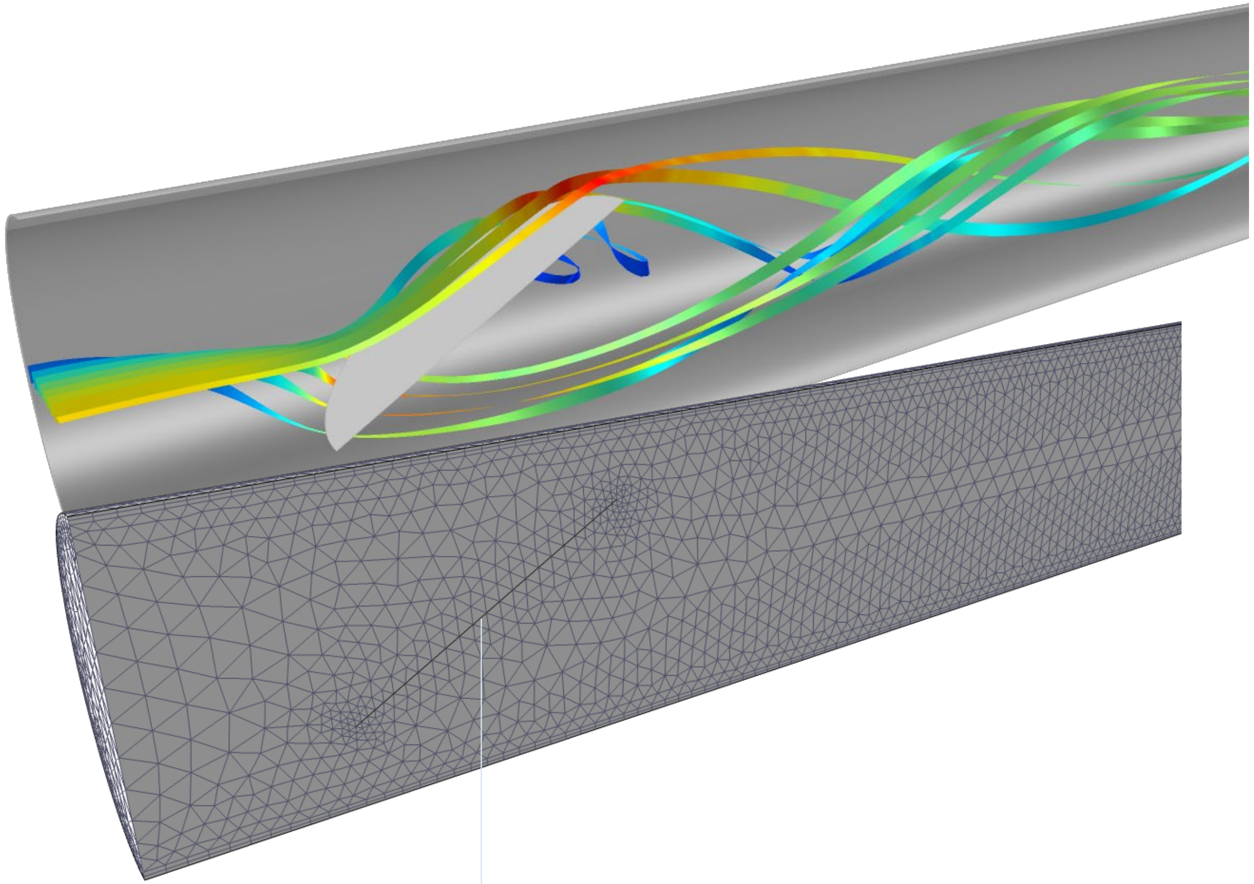




Flow in a semicircular pipe with and without the Fully Developed Flow *option*

Interior Wall

- Avoid meshing of thin structures by using interior boundaries
- Slip, no slip, moving wall
- Allows discontinuities (velocity, pressure) across the boundary



Modeled as surfaces in 3D and edges in 2D

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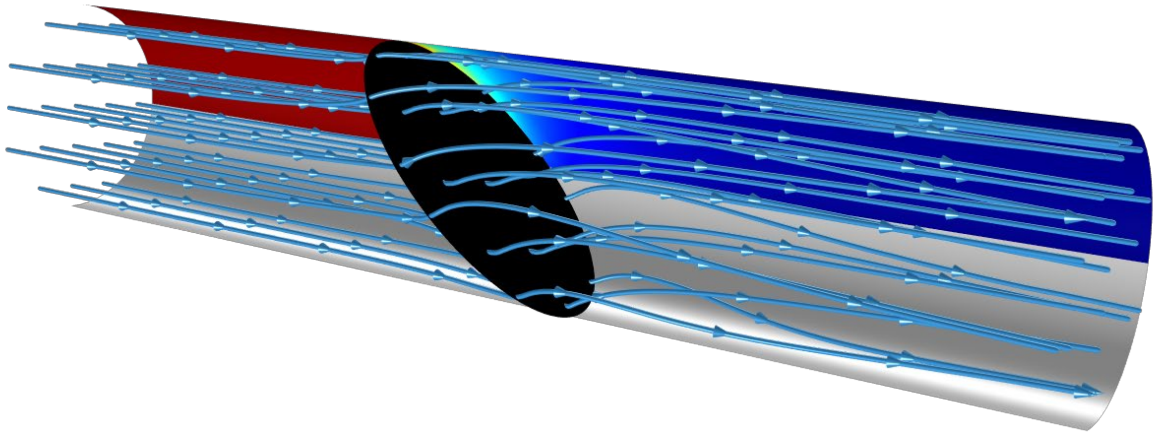
Screen

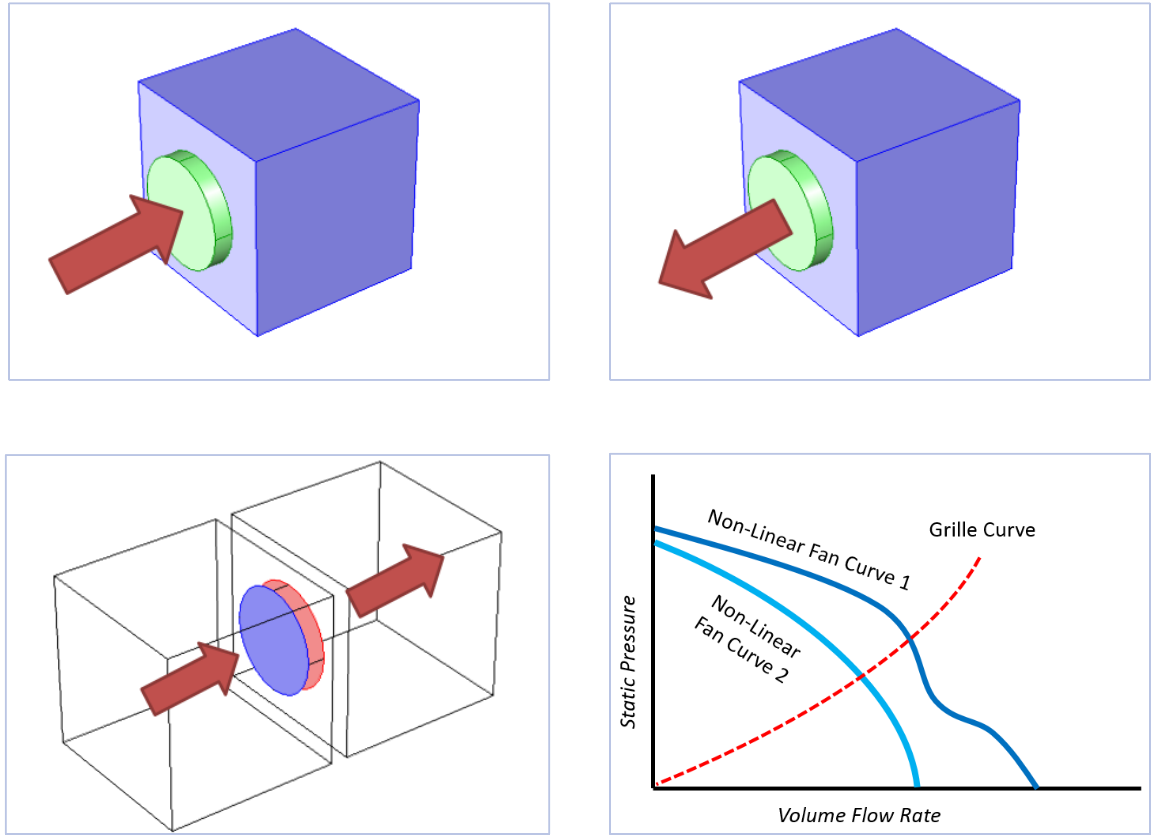
- “Screen” refers to a barrier with perforations, e.g. wire gauze, grille, or perforated plate
- Assumed to have a width, which is small compared to the resolved length scales:
  - Can be modeled as an interior edge (2D) or interior surface (3D)
- Common correlations for resistance and refraction coefficients are included



Pressure (color table) and streamlines for pipe flow obstructed by a screen inclined at 45 degrees

Fan, Interior Fan, Grille, Vacuum Pump

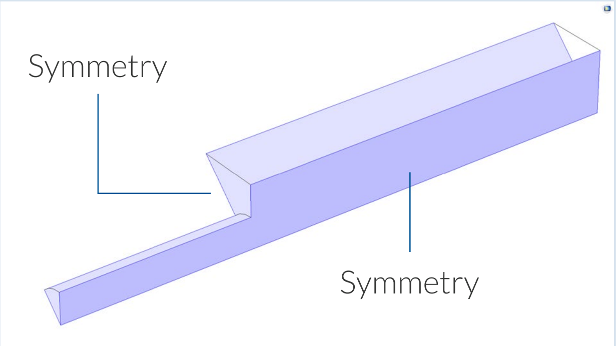
- Represented using lumped curves as boundary conditions
- Boundary can only be either an inlet or an outlet
- Fan manufacturers provide curves for the static pressure as a function of flow rate



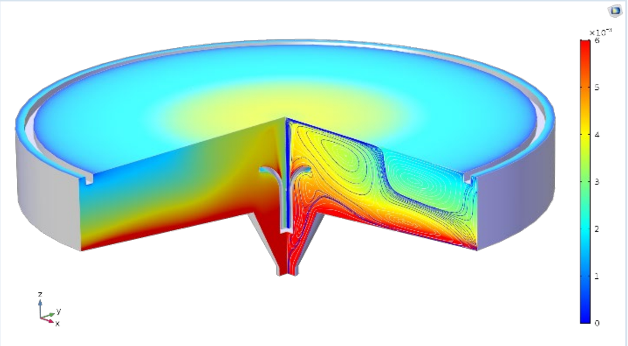
Define the flow direction on exterior and interior boundaries, and fan curve (or similar)

Symmetry, Axisymmetry, and Periodicity

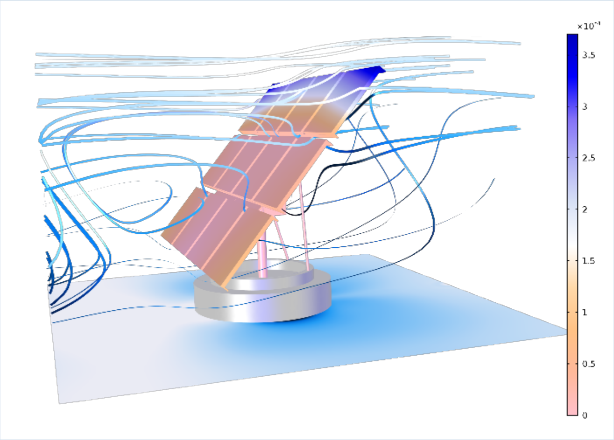
- Reduce memory requirement and computational time
- Physics, materials, and geometry must all be symmetric



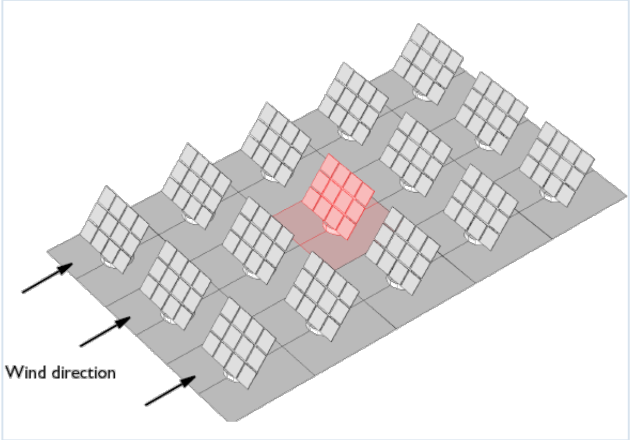
Symmetry



Axisymmetry



Periodicity



Periodicity

Although the physics, materials, and geometry are symmetric, there may be asymmetric solutions. Here is a classical example of the Karman vortex street. There is a symmetric but very unstable solution.